AD-A045 614 CORNELL UNIV ITHACA N Y SCHOOL OF OPERATIONS RESEARC--ETC F/G 5/3
A 3-PERSON COOPERATIVE GAME IN CHARACTERISTIC FUNCTION FORM FOR--ETC(U)
AUG 77 P P SHENOY N00014-75-C-0678
UNCLASSIFIED TR-357 NL

1 OF 1
ADA045614

END
DATE
FILMED
11-77
DDC

12 9

SCHOOL
OF
OPERATIONS RESEARCH
AND
INDUSTRIAL ENGINEERING

COLLEGE OF ENGINEERING
CORNELL UNIVERSITY
ITHACA, NEW YORK 14853

SCHOOL OF OPERATIONS RESEARCH
AND INDUSTRIAL ENGINEERING
COLLEGE OF ENGINEERING
CORNELL UNIVERSITY
ITHACA, NEW YORK

TECHNICAL REPORT NO. 357

August 1977

A 3-PERSON COOPERATIVE GAME IN CHARACTERISTIC FUNCTION FORM FORMULATION OF THE WORLD OIL MARKET

by

Prakash P. Shenoy

This research was supported in part by the Office of Naval Research under Contract N00014-75-C-0678 and the National Science Foundation under Grants MPS75-02024.

1. Introduction.

In the winter of 1973, some major oil exporting countries joined hands together and declared an embargo on oil exports to some western countries for political reasons. Elated by their success and the realization that they controlled a major share of the oil exports, they subsequently raised the price of oil four-fold and cut back production obtaining (in the face of an almost inelastic demand) increased revenues.

The major oil importing countries have been trying to work out an optimal policy designed to obtain their energy needs at lowest possible prices. One of the strategies considered by these countries is to attempt to split up the oil cartel by bilateral dealings or by trying to play one member country off against another. This paper analyzes the feasibility of such a strategy and its cost in financial terms using the theory of n-person cooperative games.

In Shenoy [29], the world oil market is modelled as a 2-person non-zero-sum game with the oil importing countries denoted by OPIC as one player and the oil exporting countries denoted by OPEC as the second player. In this paper, we will divide OPEC into two groups - one led by Saudi Arabia (SA) and the other led by Iran (IR). Despite many common characteristics of the two groups, each group displays different national attributes and long term commercial interests. IR, with a larger population, relatively small petroleum reserves, aggressive plans for economic development and military build-up, can use all the revenue available through major price increases. SA on the other hand, has a very small population and hence little capital absorption capability, large petroleum reserves and enormous financial reserves. In a period of rapid inflation,

SA would prefer to have oil in the ground rather than increase production. And SA also would prefer to keep prices below the substitution threshold for new energy sources because of the fear that a flood of new energy will drive the price downward substantially in advance of the time SA's petroleum reserves are exhausted. Although huge lag time of seven years and more are involved in energy substitution, SA fears the impact of large potential economies of scale in coal liquefication and other related techniques and the possibility of a significant break-through in terms of the leaning curve, all of which would help to bring down the future price of energy. IR, facing a much shorter time horizon for the exhaustion of its energy reserves, can push the price of crude oil very high without much fear of the consequences from accelerating new discoveries and the innovation of new sources of energy (see Thrall, Doran, Owen, Wall and Young [31]).

2. The Models.

The world oil market is modelled as a 3-person cooperative game with and without side payments, in characteristic function form. The characteristic function form of the game is chosen because it focuses on the bargaining process and allocation of payoffs among the players.

Player 1 called OPIC represents all the oil importing countries. Here we assume that all the major oil importing countries have formed into a cartel and bargain collectively as one unit. Player 2 called IR and player 3 called SA represent the two groups in the OPEC cartel that have between them all the oil exported to OPIC, who we assume is the sole market for the oil exports.

We shall assume that OPIC needs a total of I million barrels of oil daily (mmbd) assuming consumption required for a maximum growth of their

economy. A part of this requirement can be met by domestic production of oil. By a large investment, the domestic production of oil can be increased by finding new sources or just by working the existing wells harder using improved technology. Alternatively, the demand for oil can partly be satisfied by other fuels such as coal, nuclear fission, shale oil and other new sources that could be developed by large investment in research and development. Furthermore, the consumption of oil could be reduced by voluntary or mandatory methods such as rationing the supply of oil, an energy tax, etc. This may however, result in losses in the nation's economy. In short, the strategy for OPIC is to decide the quantity of oil imports. More formally, we will denote the strategy space of OPIC by

$$\Sigma_1 = \{x_1 \in E^1 : 0 \leq x_1 \leq I\}.$$

Associated with a strategy $x_1 \in \Sigma_1$ is a monetary cost to OPIC, denoted by $f_1(x_1)$ for restricting its imports to x_1 mmbd. $f_1(x_1)$ does not include the cost of imports. A sketch of a method of computing $f_1(x_1)$ is as follows.

Let $h(y)$ denote the total cost in million dollars daily (mm\$d) to ensure that domestic production of oil is at least y mmbd. Let $g(z)$ denote the loss in mm\$d in OPIC's G.N.P.† if the total oil (energy) consumption is restricted to z mmbd. Then we have

$$f_1(x_1) = \min_{0 \leq y \leq I - x_1} [h(y) + g(y + x)]$$

† Gross National Product. Other indicators of a nation's economy can also be used.

We will assume that $f_1(x_1)$ is a non-increasing positive real-valued, function defined on the strategy space Σ_1 of OPIC. Several studies have been made to determine the function $f_1(x_1)$ for the case of United States alone. See Shenoy [29] for more details. A sketch of the possible nature of $f_1(x_1)$ is indicated in Figure 1.

Let C_2 and C_3 denote the production capacities in million barrels of oil daily of IR and SA respectively. Let e_2 and e_3 denote the extraction cost in dollars per barrel of oil ($/b) for IR and SA rsspectiveyy. Also let M_2 and M_3 denote the capital investment in million dollars daily (mm$d), necessary to achieve a maximum growth rate of IR's and SA's economy. for $0 \leq y \leq M_2$ and $0 \leq z \leq M_3$, let $f_2(y)$ and $f_3(z)$ denote the losses in mm$d to IR's and SA's economy if capital investment is restricted to y and z mm$d respectively. Any capital in excess of M_2 and M_3 is available as capital reserves. We will assume that f_2 and f_3 are non-increasing real-valued function defined on the real interval $[0,\infty)$. A sketch of the possible nature of these functions is shown in Figure 2.

The strategy for both IR and SA is to decide on the quantity and price of oil exported.

Let

$$\Sigma = \{ (x_1, p_2, x_2, p_3, x_3) : 0 \leq x_1 \leq I \quad e_2 \leq p_2 < \infty, \; e_3 \leq p_3 < \infty, \quad 0 \leq x_2 \leq C_2, \; 0 \leq x_3 \leq C_3 \text{ and } x_1 = x_2 + x_3 \}$$

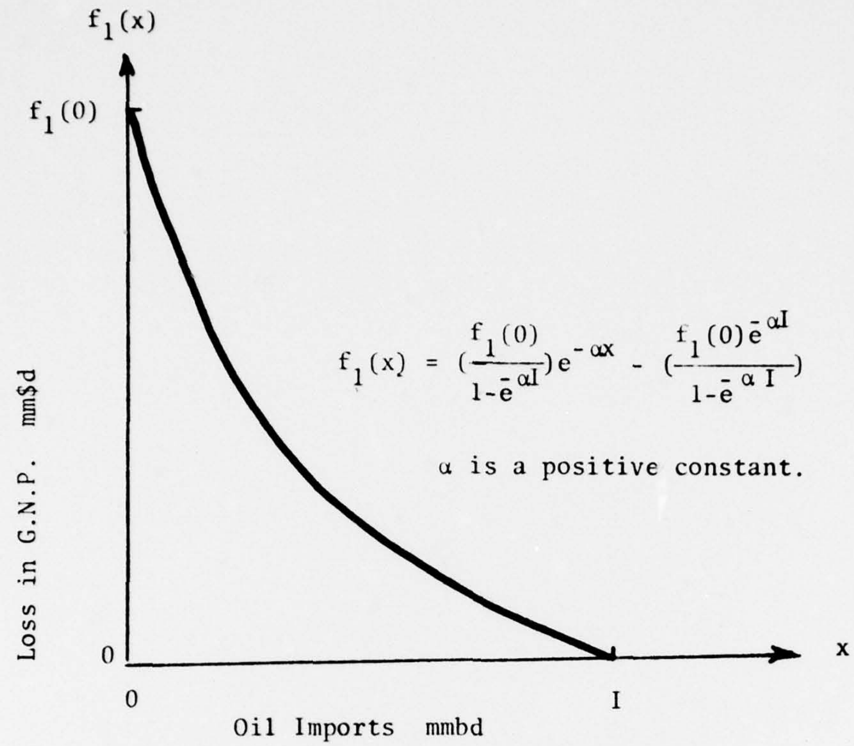


Figure 1. A sketch of the possible nature of function f_1.

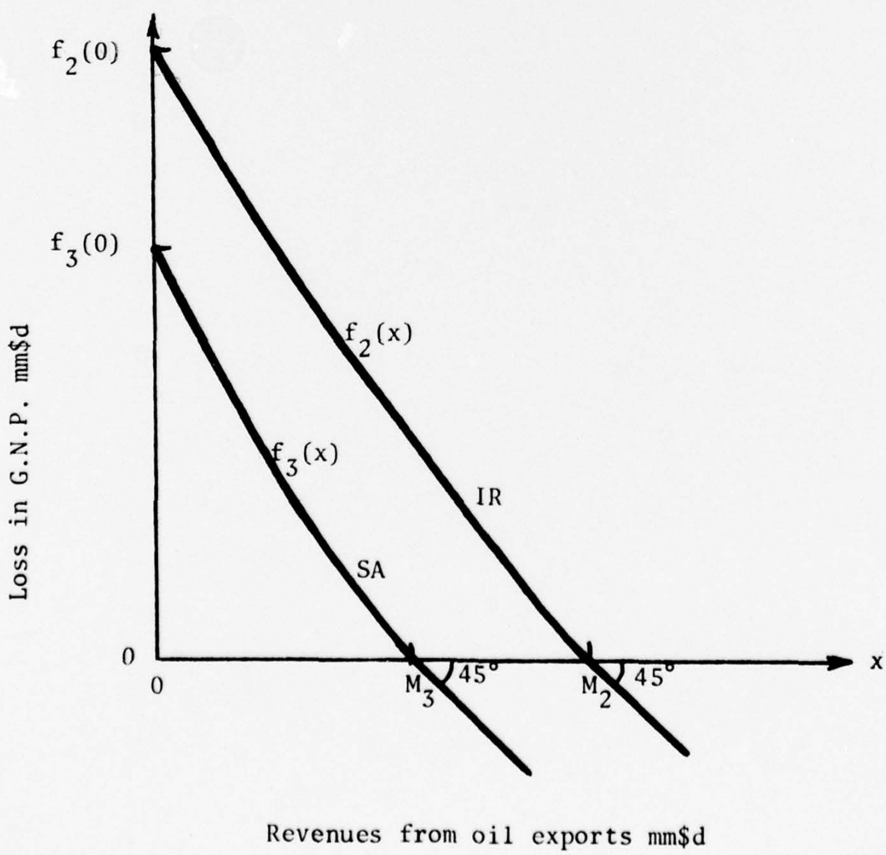


Figure 2. A sketch of the possible nature of functions f_2 and f_3.

denote the set of all possible outcomes.

For each outcome in $\sum$, there results a monetary payoff to each player. Let $A_i: \sum \to E^1$ denote the (monetary) payoff function of player i (i = 1,2,3). Then we define

$$A_1(x_1,p_2,x_2,p_3,x_3) = -f_1(x_1) - p_2 \cdot x_2 - p_3 \cdot x_3,$$
$$A_2(x_1,p_2,x_2,p_3,x_3) = -f_2((p_2-e_2) \cdot x_2), \quad \text{and}$$
$$A_3(x_1,p_2,x_2,p_3,x_3) = -f_3((p_3-e_3) \cdot x_3) .$$

Before defining the characteristic functions of the side payment and the non-side payment game. We will make the following assumptions regarding the parameters of the problem.

A.1. $C_2 < C_3 < I < C_2 + C_3$

A.2. $f_3(0) < f_2(0) < f_1(0)$

A.3. $M_3 < M_2$

A.4. $e_2 < |f'(x_1)| \quad 0 \leq x_1 \leq I$

A.5. $e_3 < |f'(x_1)| \quad 0 \leq x_1 \leq I$

A.6. $|f_2'(x_2)| > 1 \quad 0 \leq x_2 \leq M_2$

A.7. $|f_3'(x_3)| > 1 \quad 0 \leq x_3 \leq M_3$.

Assumptions A.1 - A.7 represent the realities of the situation being modelled.

3. The Side Payment Model.

In this section, we will assume that unrestricted side payments are allowed. We will use the von Neumann-Morgenstern model of the characteristic

function (See von Neumann and Morgenstern [23]). This is derived by considering the maximum each coalition can guarantee itself under any circumstances. Also, we assume that utility is linear in money.

Let $N = \{1,2,3\}$ denote the set of players, 2^N, the set of all subsets of N called coalitions and $v\colon 2^N \to E^1$, the characteristic function which is defined as follows.

$$v(\{\emptyset\}) = 0, \quad v(\{1\}) = -f_1(0),$$

$$v(\{2\}) = -f_2(0), \quad v(\{3\}) = -f_3(0),$$

$$v(\{1,2\}) = \max_{\substack{0 \le x_2 \le c_2 \\ e_1 \le p_1 < \infty}} \{-f_1(x_1) - p_2 \cdot x_1 - f_2((p_2 - e_2)\cdot x_1)\},$$

$$v(\{2,3\}) = -f_2(0) - f_3(0),$$

$$v(\{1,3\}) = \max_{\substack{0 \le x_1 \le c_3 \\ e_3 \le p_3 < \infty}} \{-f_1(x_1) - p_3 \cdot x_1 - f_3((p_3 - e_3)\cdot x_1)\}$$

and

$$v(\{1,2,3\}) = \max_{\substack{0 \le x_2 \le c_2 \\ 0 \le x_3 \le c_3 \\ e_2 \le p_2 < \infty \\ e_3 \le p_3 < \infty}} \{-f_1(x_2+x_3) - p_2 \cdot x_2 - p_3 \cdot x_3 - f_2((p_2 - e_2)\cdot x_2) - f_3((p_3 - e_3)\cdot x_3)\} .$$

We shall now determine the relative magnitudes of the values of the characteristic function. We have†

---

†To condense notation, we shall drop the parenthesis around the players in a coalition and denote, for example, $v(\{1,2\})$ by $v(12)$.

$$v(12) = \max_{\substack{0 \le x_1 \le C_1 \\ e_2 \le p_2 < \infty}} \{-f_1(x_1) - p_2 \cdot x_1 - f_2((p_2 - e_2) \cdot x_1)\}$$

Clearly, by Assumptions A.4 and A.6, the maximum in the above expression is achieved at $x_1 = C_2$, $p_2 = e_2 + \frac{M_2}{C_2}$, and so we get

$$\begin{aligned} v(12) &= -f_1(C_2) - e_2 \cdot C_2 - M_2 - f_2(M_2) \\ &= -f_1(C_2) - e_2 \cdot C_2 - M_2 \end{aligned}$$

Similarly, we get

$$v(13) = -f_1(C_3) - e_3 \cdot C_3 - M_3$$

and

$$\begin{aligned} v(123) &= -f_1(I) - K - M_1 - M_2 \\ &= -K - M_1 - M_2 \end{aligned}$$

where $K = \min\ e_2 \cdot C_2 + e_3 \cdot (I - C_2),\ e_3 \cdot C_3 + e_2 \cdot (I - C_3)$.

In 0-normalized form, the characteristic function is as follows.

$v(\emptyset) = v(1) = v(2) = v(3) = 0,$

$v(12) = f_1(0) + f_2(0) - f_1(C_2) - e_2 \cdot C_2 - M_2,$

$v(13) = f_1(0) + f_3(0) - f_1(C_3) - e_3 \cdot C_3 - M_3,$

$v(23) = 0,$

and

$v(123) = f_1(0) + f_2(0) + f_3(0) - K - M_2 - M_3.$

we make two additional assumptions as follows.

A.8. $f_3(0) - f_1(C_3) - e_3 \cdot C_3 - M_3 > f_2(0) - f_1(C_2) - e_2 \cdot C_2 - M_2$

A.9. $f_2(0) - K - M_2 > -f_1(C_3) - e_3 \cdot C_3$.

So we have the following relation.

$$0 < v(12) < v(13) < v(123).$$

4. <u>The Non-Side Payment Model.</u>

We consider cooperative games without side payments because it approximates the real life situation more closely than games with side payments. Although side payments are legal, utility is usually nonlinear in money and this results in a situation not covered by the von Neumann-Morgenstern theory. (See Aumann [2,4]).

Before defining the characteristic function of the non-side payment game, we will define the utility function of each of the players. Each monetary payoff has a particular utility to each player. Let $u_i: A_i(\Sigma) \to [0,1]$ denote the utility function of player i $(i = 1,2,3)$. Define:

$$u_1(z) = \begin{cases} \dfrac{f_1(0) + z}{f_1(0) - K \cdot I} & \text{if } -f_1(0) \leq z \leq -K \cdot I \\ 1 & \text{if } z \geq -K \cdot I \end{cases}$$

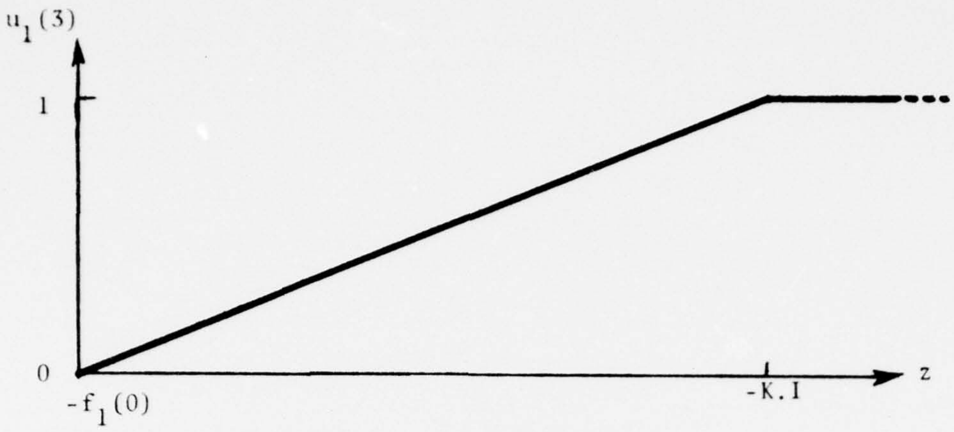


Figure 3. The utility function of player 1.

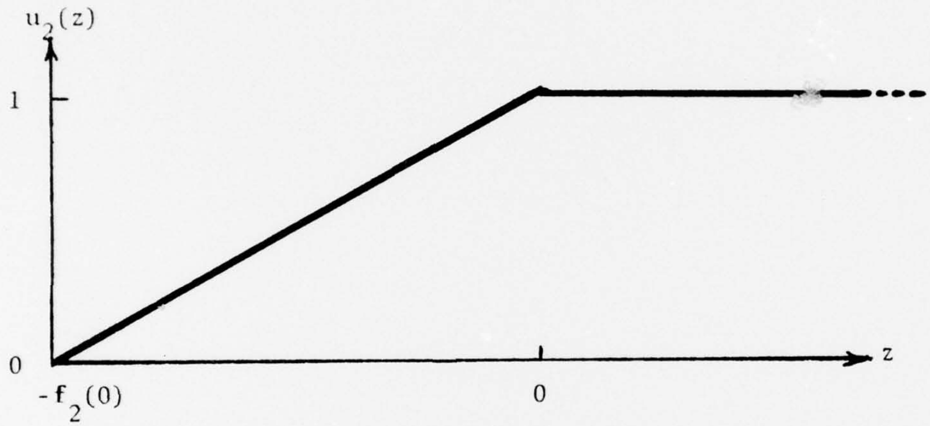


Figure 4. The utility function of player 2.

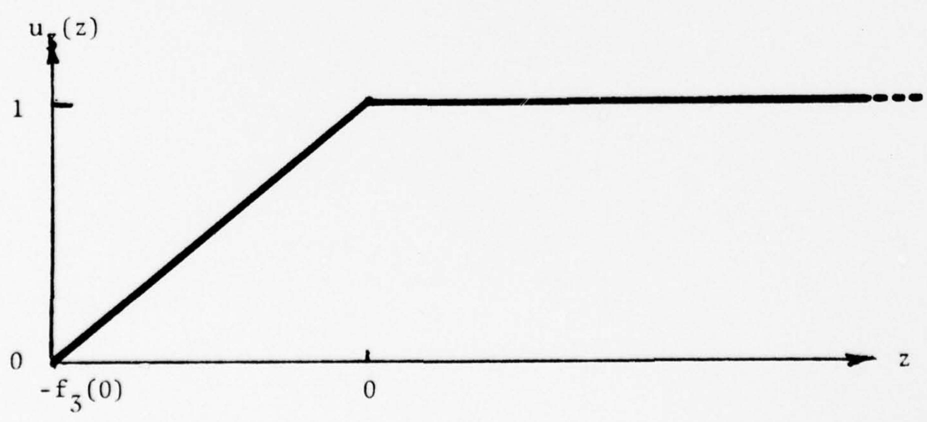


Figure 5. The utility function of player 3.

$$u_2(z) = \begin{cases} \dfrac{f_2(0) + z}{f_2(0)} & \text{if } -f_2(0) \le z \le 0 \\ 1 & \text{if } z \ge 0 \end{cases}$$

$$u_3(z) = \begin{cases} \dfrac{f_3(0) + z}{f_3(0)} & \text{if } -f_3(0) \le z \le 0 \\ 1 & \text{if } z \ge 0 \end{cases}$$

We can now define the characteristic function of the non-side payment game. Denote by E^S the subspace of E^3 spanned by the axes belonging to the players in a subset $S \subset N$. The characteristic function $V: 2^N \to E^3$ associates with each coalition $S \subset N$, a subset $V(S)$ of E^S. Intuitively, $V(S)$ represents the set of payoff vectors that coalition S can guarantee itself. Let E^3_+ denote the positive orthant of E^3. Also let $\mathrm{Conv}\{a_1,\ldots,a_p\}$ denote the convex hull of the vectors in $\{a_1,\ldots,a_p\}$.

We define V as follows.

$$\begin{aligned}
V(\emptyset) &= E^3_+ \\
V(1) &= (u_1(-f_1(0)),\ 0,\ 0) - E^3_+ \\
&= (0,0,0) - E^3_+ \\
V(2) &= (0,\ u_2(-f_2(0)),\ 0) - E^3_+ \\
&= (0,0,0) - E^3_+ \\
V(3) &= (0,0,u_3(-f_3(0))) - E^3_+ \\
&= (0,0,0) - E^3_+ \\
V(12) &= \mathrm{Conv}\{(u_1(-f_1(C_2) - e_2 \cdot C_2 - M_2),\ u_2(-f_2(M_2)),\ 0), \\
&\qquad (u_1(-f_1(C_2) - e_2 \cdot C_2),\ u_2(-f_2(0)),\ 0)\} - E^3_+
\end{aligned}$$

$$= \mathrm{Conv}\,\{(u_1(-f_1(C_2) - e_2\cdot C_2 - M_2),\ 1,0),$$
$$(u_1(-f_1(C_2) - e_2\cdot C_2),\ 0,0)\} - E^3_+.$$

$$V(13) = \mathrm{Conv}\{(u_1(-f_1(C_3) - e_3\cdot C_3 - M_3),\ 0,\ u_3(-f_3(M_3))),$$
$$(u_1(-f_1(C_3) - e_3\cdot C_3),\ 0,\ u_3(-f_3(0)))\} - E^3_+$$
$$= \mathrm{Conv}\{(u_1(-f_1(C_3) - e_3\cdot C_3 - M_3),\ 0,1),$$
$$(u_1(-f_1(C_3) - e_3\cdot C_3),\ 0,0)\} - E^3_+.$$

$$V(23) = (0,\ u_2(-f_2(0)),\ u_3(-f_3(0))) - E^3_+$$
$$= (0,0,0) - E^3_+.$$

$$V(123) = \mathrm{Conv}\{(u_1(-f_1(I) - K\cdot I),\ u_2(-f_2(0)),\ u_3(-f_3(0))),$$
$$(u_1(-f_1(I) - K\cdot I - M_2),\ u_2(-f_2(M_2)),\ u_3(-f_3(0))),$$
$$(u_1(-f_1(I) - K\cdot I - M_3),\ u_2(-f_2(0)),\ u_3(-f_3(M_3))),$$
$$(u_1(-f_1(I) - K\cdot I - M_2 - M_3),\ u_2(-f_2(M_2)),\ u_3(-f_3(M_3)))\}$$
$$- E^3_+$$
$$= \mathrm{Conv}\{(1,0,0),\ (u_1(-f_1(I) - K\cdot I - M_2),\ 1,0),$$
$$(u_1(-f_1(I) - K\cdot I - M_3),\ 0,1),$$
$$(u_1(-f_1(I) - K\cdot I - M_2 - M_3),\ 1,1)\} - E^3_+ .$$

Here we assume that

A.10. $-f_1(I) - K\cdot I - M_2 - M_3 > -f_1(0)$.

This completes the formulation of the world oil market as a non-side payment game. In the subsequent sections, we study the solutions of the side payment and the non-side payment game.

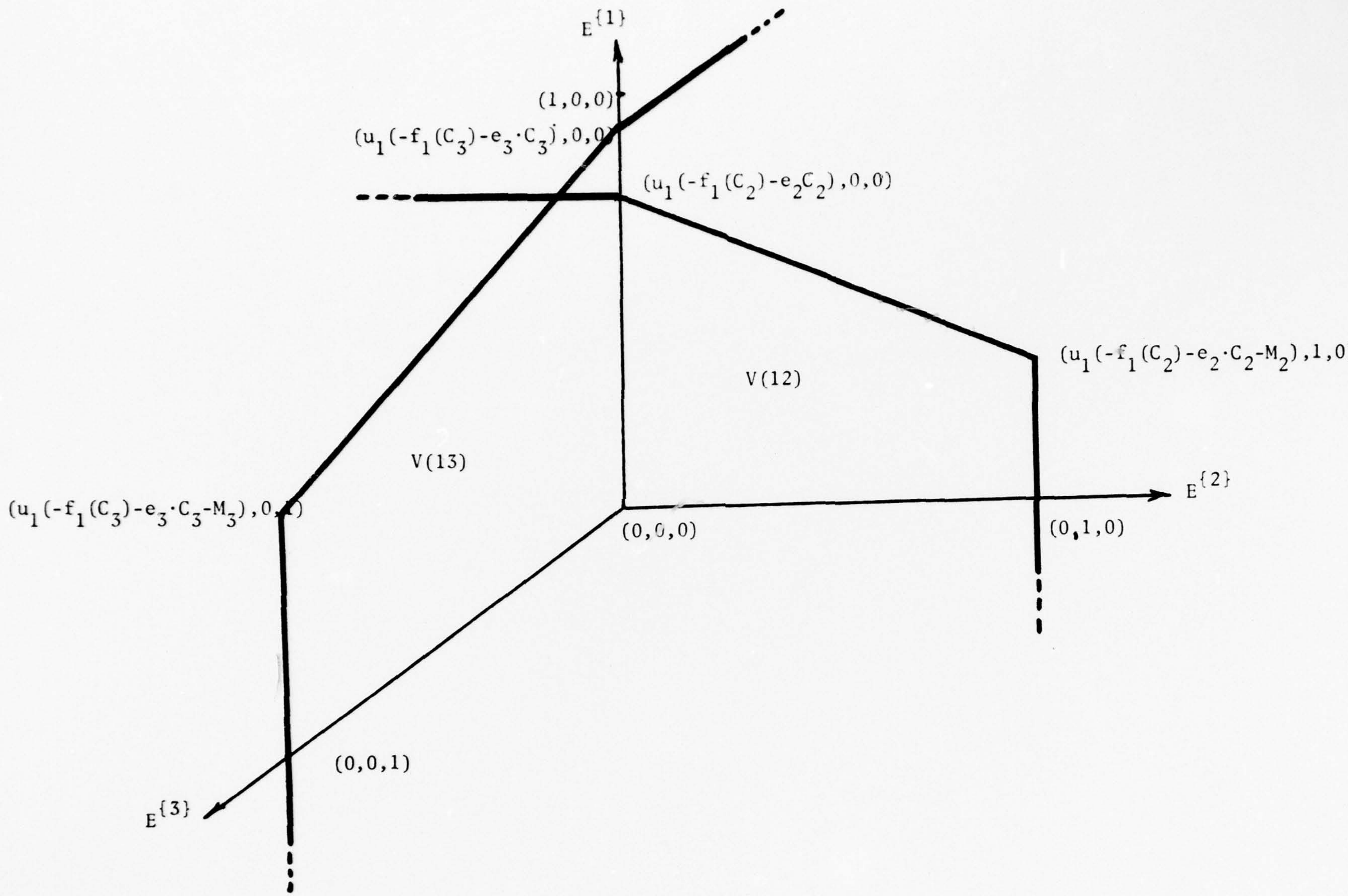


Figure 6. Geometrical representation of V(12) and V(13).

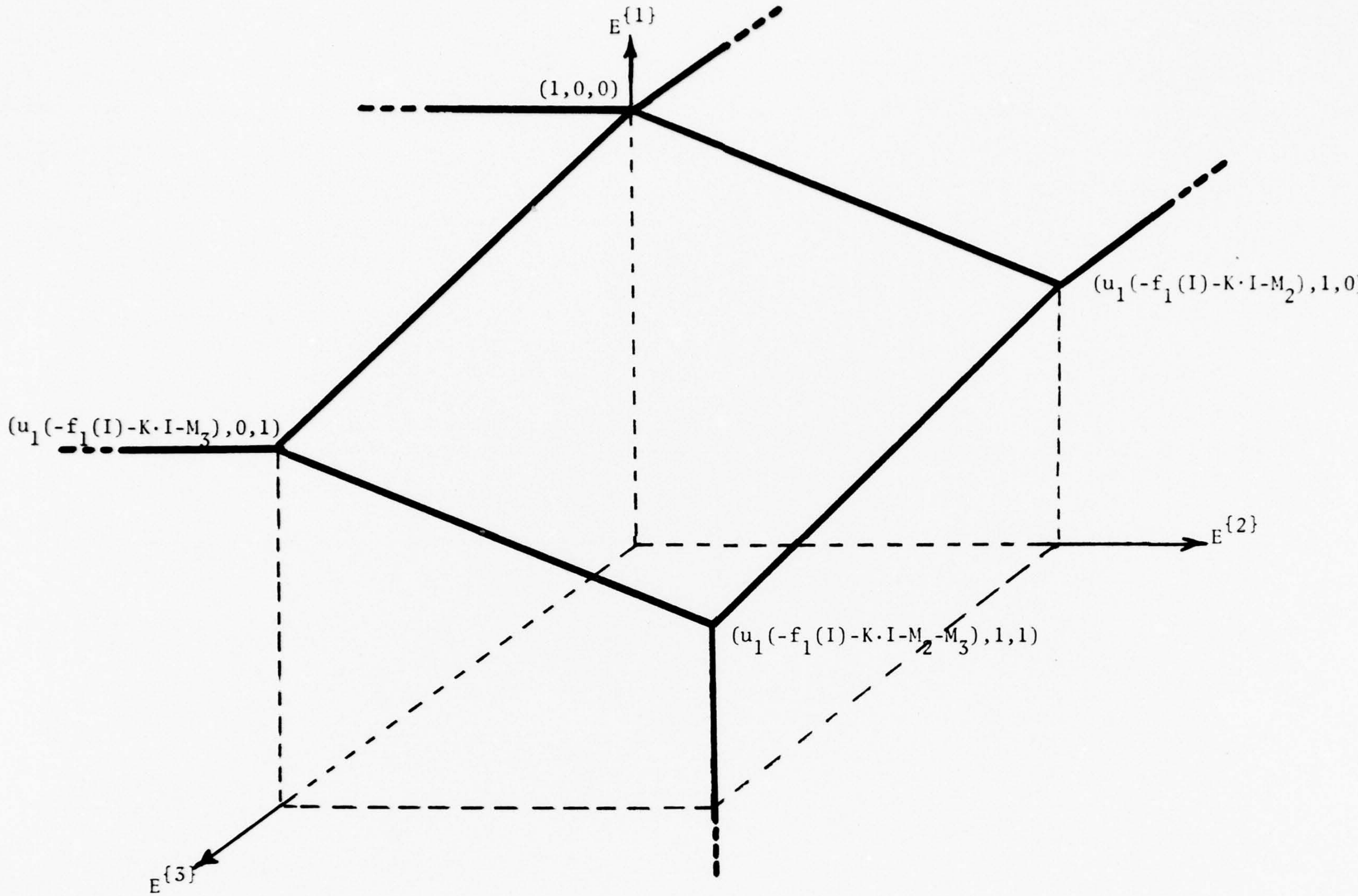


Figure 7. Geometrical representation of V(123).

5. Solutions of the Side Payment Game.

There are many solution concepts for n-person cooperative games with side payments. Each solution has its own intuitive justification. In this section we will study the core, the Shapley value, the bargaining set and the nucleolus.

Let us denote the characteristic function defined in Section 3 as follows.

$$v(123) = \gamma, \quad v(13) = \beta, \quad v(12) = \alpha$$

where $\gamma > \beta > \alpha$.

5.1. The Core.

The core of a game with side payments was first studied by Gillies [15] and Shapley. An imputation in this game is any vecoor (y_1,y_2,y_3) such that

$$y_1 \geq v(1), \quad y_2 \geq v(2), \quad y_3 \geq v(3) \quad \text{(individual rationality)}$$

and

$$y_1 + y_2 + y_3 = v(123) \quad \text{(group rationality)}$$

y_1,y_2 and y_3 represent possible payoffs to players 1,2, and 3 respectively. The core of our game consists of those imputations (if any) which satisfy the following relations.

$$y_1 + y_2 \geq \alpha, \quad y_1 + y_3 \geq \beta, \quad y_2 + y_3 \geq 0.$$

Let Co denote the core of our game. Then it is given as follows:

Case (i). $\beta < \gamma \leq \alpha + \beta$.

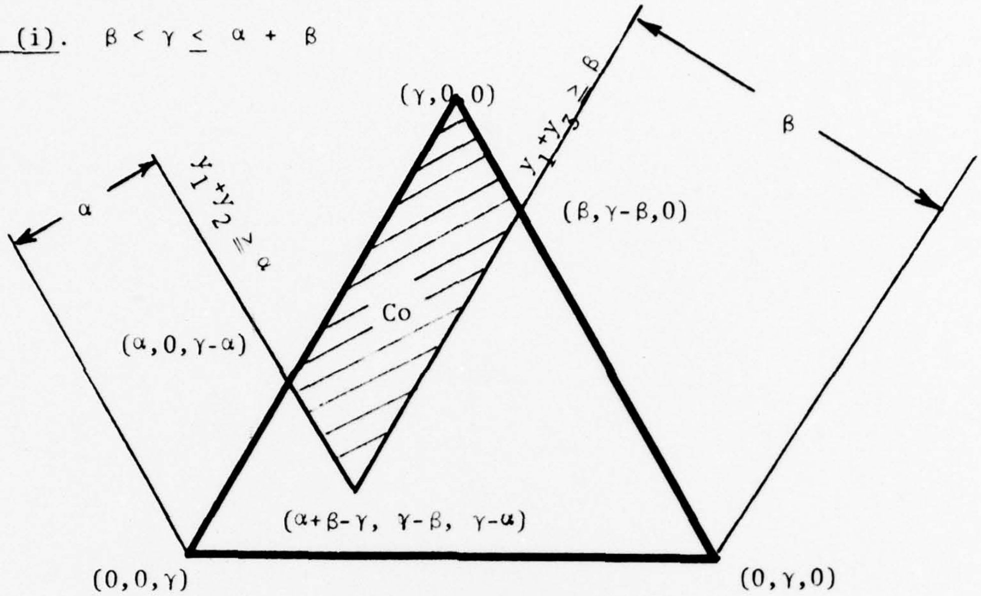
$$Co = \mathrm{Conv}\{(\gamma,0,0), \ (\beta,\gamma-\beta,0), \ (\alpha+\beta-\gamma, \ \gamma-\beta, \ \gamma-\alpha), \ (\alpha,0,\gamma-\alpha)\}$$

Case (ii). $\gamma > \alpha + \beta$

$$Co = \mathrm{Conv}\{(\gamma,0,0), \ (\beta,\gamma-\beta,0), \ (0,\gamma-\beta,\beta), \ (0,\alpha,\gamma-\alpha), \ (\alpha,0,\gamma-\alpha)\}.$$

(See Figure 5.)

Case (i). $\beta < \gamma \leq \alpha + \beta$



Case (ii). $\gamma > \alpha + \beta$

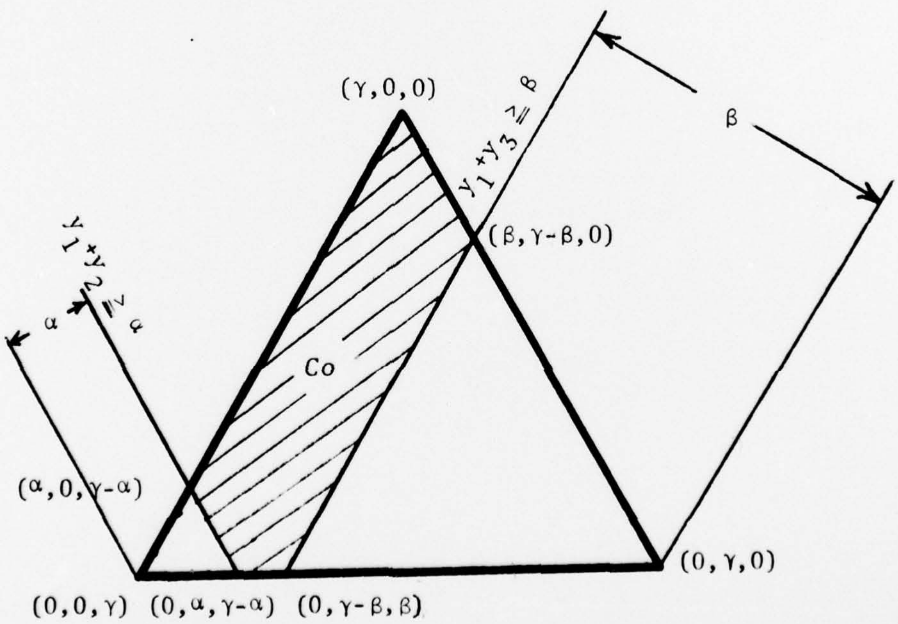


Figure 8. Representation of the core in barycentric coordinates .

The outcomes in the core have to be interpreted carefully. The core as defined above assumes that IR and SA (players 2 and 3) are acting independently without any collusion between themselves. Also we assume here that all the oil consumers are acting together as one player. (These assumptions are not based on reality but describe a scenario where OPEC splits up into two and the oil consuming countries form a cartel). In this situation, we have a market with one buyer (OPIC) and two sellers (IR and SA). As would be expected from intuitive considerations, OPIC is at an advantage since he can play one seller off against another. The outcomes in the core reflect this fact. Also the core indicates that SA is in a relatively better position compared to IR. This is also very intuitive as SA has more oil than IR and also has a lesser need for revenues compared to IR. The core consists of many outcomes and does not distinguish any particular imputation as more likely than others.

5.2. The Shapley value.

The rationale for the Shapley value is in terms of the bargaining power which each player imagines he possesses. This power (as estimated by the player in question) is based on what his joining each coalition contributes to that coalition.

For a 3-person game, the Shapley value, denoted by (ϕ_1, ϕ_2, ϕ_3) is as follows

$$\phi_1 = 1/3(v(123) - v(23)) + 1/6(v(12) - v(2)) + 1/6(v(13) - v(3)) + 1/3(v(1) - v(\phi)).$$

$$\phi_2 = 1/3(v(123) - v(13)) + 1/6(v(12) - v(1)) + 1/6(v(23) - v(3)) + 1/3(v(2) - v(\phi)).$$

$$\varphi_3 = 1/3(v(123) - v(12)) + 1/6(v(13) - v(1)) + 1/6(v(23) - v(2)) + 1/3(v(3) - v(\phi)).$$

Substituting the values of the characteristic function in the above expressions, we get

$$\phi_1 = (2\gamma + \alpha + \beta)/6$$
$$\phi_2 = (2\gamma + \alpha - 2\beta)/6$$
$$\phi_3 = (2\gamma - 2\alpha + \beta)/6$$

Note that $\phi_1 + \varphi_2 + \phi_3 = \gamma$. Also since $\gamma > \beta > \alpha$, we have

$$\phi_1 > \phi_3 > \varphi_2$$

The Shapley value also indicates that OPIC has an advantage over IR and SA and SA has an edge over IR. The Shapley value besides determining a unique allocation of the payoff solely by the characteristic function of the game, has built into it a certain equity principle. This solution might therefore be a strong contender for the status of a "normative" solution, i.e., one which "rational players" ought to accept. It's weakness is that it derives entirely from the characteristic function of the game and not from what is "beneath" the characteristic function, i.e, the strategic structure of the game itself rather than the bargaining positions of the players in the process of coalition formation.

5.3. The Bargaining Set $M_1^{(i)}$.

The bargaining set was first introudced by Aumann and Maschler (A-M) [5]. The A-M bargaining set was developed to attack the following general question:

If the players in a cooperative n-person game have decided upon a specific coalition structure, how then will they distribute among themselves the values of the various coalitions in such a way that some stability requirements will be satisfied (cf. Davis and Maschler [11, p. 39]). These stability requirements are based on the idea that a "stable" payoff configuration should offer some security in the sense that each "objection" could be met by a "counter-objection". Several kinds of bargaining sets were defined. One of these denoted by $M_1^{(i)}$ was shown by Peleg [25] to be nonempty for each partitioning of the players into a coalition structure.

The bargaining set $M_1^{(i)}$ for our game is given by:

$$M_1^{(i)}(P) = \begin{cases} (0,0,0) & \text{if} \quad P = (1)(2)(3)^{\dagger} \\ (\alpha,0,0) & \text{if} \quad P = (12)(3) \\ (\alpha \leq y_1 \leq \beta, 0, \beta - y_1)^{\dagger\dagger} & \text{if} \quad P = (13)(2) \\ (0,0,0) & \text{if} \quad P = (1)(23) \\ \text{Co} & \text{if} \quad P = (123). \end{cases}$$

The bargaining set corresponding to the grand coalition coincides with the core. The bargaining set also indicates that when OPIC and IR are in a coalition against SA. IR has no bargaining power at all vis-a-vis OPIC. An observation of all the outcomes in the bargaining set reveals that it is in the mutual interest of all the players to form the grand coalition (consisting of all the 3 players).

5.4. The Nuclelolus and the Normalized-Nucleolous.

The nuclelolus, ν, was defined by Schmeidler [27]. Let $y = (y_1, y_2, y_3)$ be an imputation. Then the excess of coalition R with respect to imputation y is

---

†For convenience of notation, the partition $\{\{1\},\{2\},\{3\}\}$ is denoted by (1)(2)(3), etc.

††Denotes the set $\{(y_1, 0, \beta - y_1): \alpha \leq y_1 \leq \beta\}$

$$e_R(y) = v(R) - \sum_{i \in R} y_i$$

The excess of coalition R with respect to imputation y is a measure of coalition R's "complaint" against imputation y. The nucleolus is that imputation which minimizes the "loudest complaint." (In case of a tie in the largest complaint, the next largest excesses are compared and so on.) The nucleolus consists of a unique imputation in the bargaining set $M_1^{(i)}$ and the core if the latter is nonempty

The normalized-nucleolus (n-nucleolus) μ suggested by Lucas and studied by Grotte [16] is defined in the same manner as the nucleolus except that that excesses $e_R(y)$ are replaced by normalized-excesses (n-excesses)

$$e_R^{\mu}(y) = \frac{e_R(y)}{|R|}$$

where $|R|$ denotes the cardinality of coalition R.

The nucleolus ν for our game is given as follows

Case (i). $\gamma > 3\beta$

$$\nu = (\gamma/3,\ \gamma/3,\ \gamma/3)$$

Case (ii). $\beta + 2\alpha \leq \gamma \leq 3\beta$

$$\nu = ((\gamma + \beta)/4,\ (\gamma - \beta)/2,\ (\gamma + \beta)/4)$$

Case (iii). $\alpha + \beta \leq \gamma \leq \beta + 2\alpha$

$$\nu = ((\alpha + \beta)/2,\ (\gamma - \beta)/2\ \ (\gamma - \alpha)/2)$$

Case (iv). $\beta \leq \gamma \leq \alpha + \beta$

$$\nu = ((\alpha + \beta)/2,\ (\gamma - \beta)/2,\ (\gamma - \alpha)/2)$$

The n-nucleolus μ for our game is given (in all cases) by

$$\mu = ((2\gamma + \beta + 3\alpha)/6,\ (\gamma - \beta)/3,\ (2\gamma + \beta - 3\alpha)/6).$$

If we denote $\nu = (\nu_1, \nu_2, \nu_3)$ and $\mu = (\mu_1, \mu_2, \mu_3)$ then in all cases we have

$$\nu_1 \geq \nu_2 \geq \nu_3 \text{ and } \mu_1 \geq \mu_2 \geq \mu_3.$$

6. Solutions of the Non-Side Payment Game.

In this section, we will study the core and the bargaining set of the non-side payment game defined in Section 4.

6.1. The Core.

The core of a game without side payments has been studied by Aumann [3], Billera [8,9] and Scarf [26]. A vector of utility levels is suggested which is feasible for all the players acting collectively and an arbitrary coalition is examined to see whether it can provide higher utility levels for all of its members. If this is possible, the utility vector which was originally suggested is said to be dominated by the coalition. The core of the n-person game consists of those utility vectors which are feasible for the entire group of players and which can be dominated by no coalition.

For our game, the core C is given as follows.

$$C = \mathrm{Conv}\{(1,0,0),\ (u_1(-f_1(C_3) - e_3 \cdot C_3),\ p_2, 0),$$
$$(u_1(-f_1(C_3)-e_3 \cdot C_3-M_3),\ p_2', 1),$$
$$(u_1(-f_1(I) - K \cdot I-M_3),\ 0, 1)\}$$

where
$$p_2 = \frac{1-u_1(-f_1(C_3)-e_3 \cdot C_3)}{1-u_1(-f_1(I)-K \cdot I-M_2)}$$

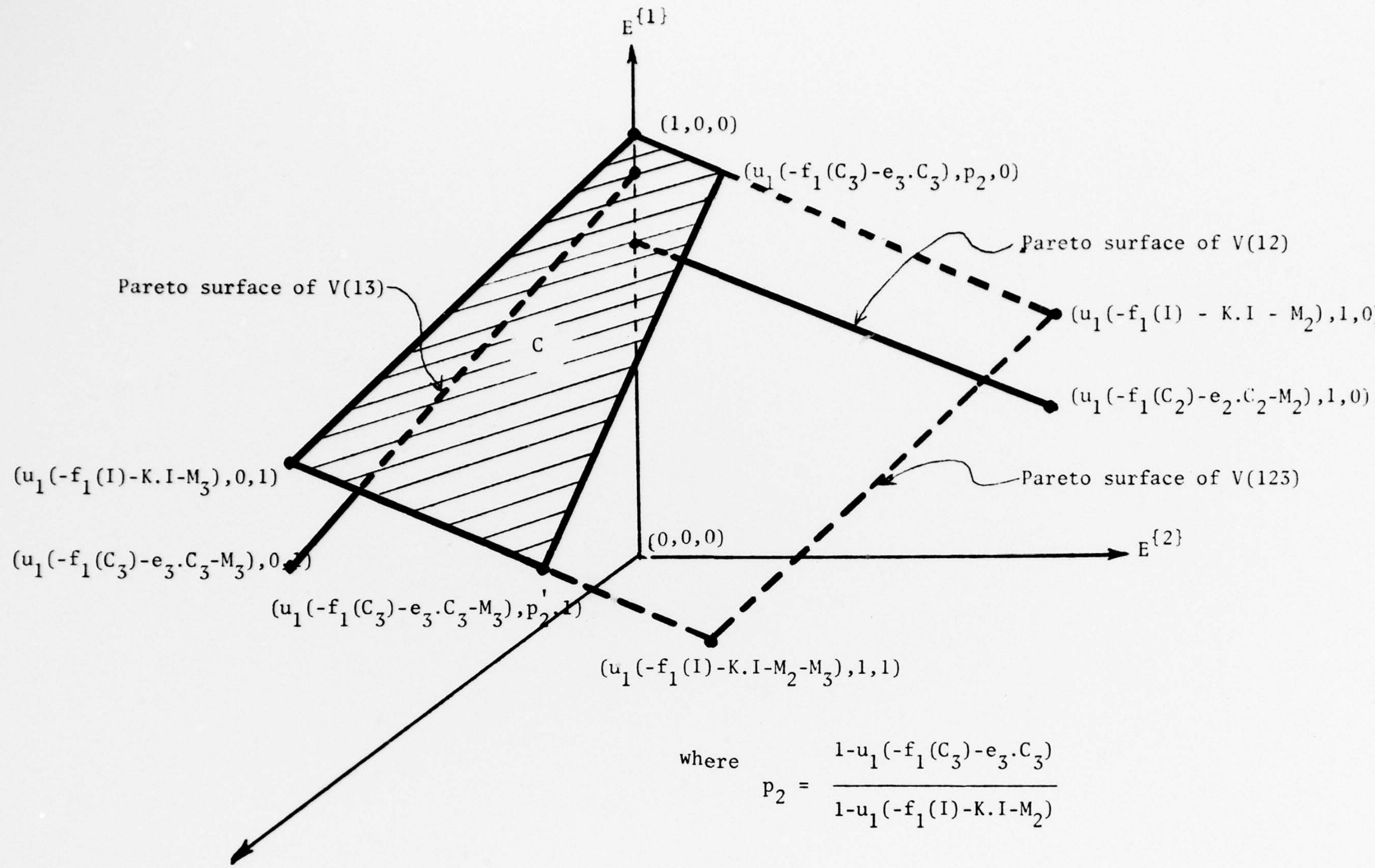
and
$$p_2' = \frac{u_1(-f_1(C_3)-e_3 \cdot C_3-M_3)-u_1(-f_1(I)-K \cdot I-M_3)}{u_1(-f_1(I)-K \cdot I-M_2-M_3)-u_1(-f_1(I)-K \cdot I-M_3)}$$

(See Figure 9).

The core again exhibits the advantage of OPIC over SA and IR and the advantage of SA over IR.

6.2. The Bargaining Set.

The A-M bargaining set $M_1^{(i)}$ was generalized by Peleg [24] to games without side payments. However he showed that it may be empty for some games. Billera [7] proposed another bargaining set $\tilde{M}_1^{(i)}$ based on the following simple principle. A payoff vector y is said to belong to the bargaining set $\tilde{M}_1^{(i)}$ if whenever play k has a justified objection (i.e. an objection that has no counterobjection) against player ℓ at y, then there exists a chain of justified objections all at y leading from player ℓ to player k (via other players) Asscher [1] proved that $\tilde{M}_1^{(i)}$ is never empty for games without side payments.



where $$p_2 = \frac{1-u_1(-f_1(C_3)-e_3.C_3)}{1-u_1(-f_1(I)-K.I-M_2)}$$

Figure 9. The core of the non side payment game.

$$p_2' = \frac{u_1(-f_1(C_3)-e_3.C_3-M_3) - u_1(-f_1(I)-K.I-M_3)}{u_1(-f_1(I)-K.I-M_2-M_3) - {}_1(-f_1(I)-K.I-M_3)}$$

For our games we have $M_1^{(i)} = \tilde{M}_1^{(i)}$ and it is given as follows (see Figure 10)

$$M_1^{(i)} = \begin{cases} (0,0,0) & \text{if } P = (1)(2)(3) \\ (u_1(-f_1(C_2)-e_2\cdot C_3),\ 0,0) & \text{if } P = (12)(3) \\ (0,0,0) & \text{if } P = (1)(23) \\ \text{Conv}\{(u_1(-f_1(C_3)-e_2\cdot C_3),\ 0,0), & \text{if } P = (13)(2) \\ \quad (u_1(-f_1(C_2)-e_2\cdot C_2),\ 0,p_3)\} & \\ C & \text{if } P = (123) \end{cases}$$

where
$$p_3 = \frac{u_1(-f_1(C_3)-e_3\cdot C_3)-u_1(-f_1(C_2)-e_2\cdot C_2)}{u_1(-f_1(C_3)-e_3\cdot C_3)-u_1(-f_1(C_3)-e_3\cdot C_3-M_3)}$$

As in the side payment case, the bargaining set for the grand coalition coincides with the core as determined in Section 6.1. Also it is observed that it is in the mutual interest of all the players to form the grand coalition.

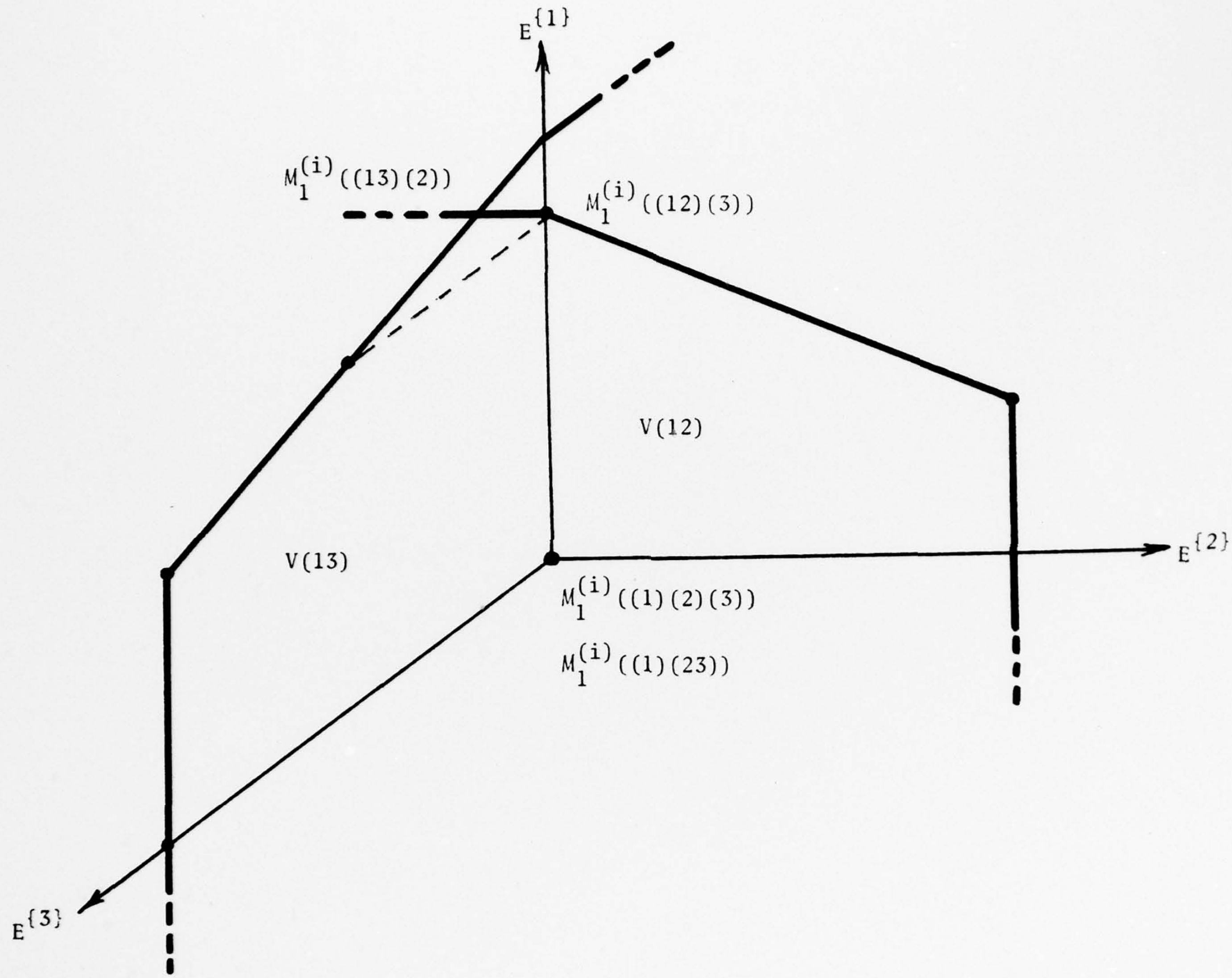


Figure 10. The bargaining set $M_1^{(i)}$ of the non-side payment game.

REFERENCES

1. N. Asscher, "An ordinal bargaining set for games without side payments", Math. of Operations Research, 1, 1976, pp. 381-389.

2. R.J. Aumann, "Linearity of unrestrictedly transferable utilities", Nav. Res. Logist. Quart, 7, 1960, pp. 281-284.

3. R.J. Aumann, "The core of a cooperative game without side payments", Trans. Amer. Math. Soc., 98, 1961, pp. 539-552.

4. R.J. Aumann, "A Survey of cooperative games with side payments," Essays in Mathematical Economics in Honor of O. Morgenstern, ed. by M. Shubik, Princeton University Press, Princeton, N.J., 1967, pp. 3-27.

5. R.J. Aumann and M. Maschler, "The bargaining set for cooperative games," Advance in Game Theory, M. Dresher, L.S. Shapley, and A.W. Tucker, eds., Annals of Mathematics Studies No. 52, Princeton University Press, Princeton, N.J., 1964, pp. 443-476.

6. J. Becker, G. Owen, and R.M. Thrall, "A game theoretic analysis of alternate source development", in Strategic Studies for Energy Planning and Management, A Final Report on N.S.F. Grant NSF SIA75-04329, Rice University, Houston, Texas, June, 1976.

7. L.J. Billera, "Existence of general bargaining sets for cooperative games without side payments", Bull. Amer. Math. Soc., 76, 1970, pp. 375-379.

8. L.J. Billera, "Some theorems on the core of an n-person game without side payments", SIAM J. Appl. Math., 18, 1970, pp. 567-579.

9. L.J. Billera, "Some recent results in n-person game theory", Math. Prog., 1, 1971, pp. 58-71.

10. C. Bird and M. Sampson, "A game theory model of O.P.E.C., oil consumers and oil companies, emphasizing coalition formation", Unpublished Manuscript, August, 1975.

11. M. Davis and M. Maschler, "Existence of stable payoff configurations for cooperative games", Essays in Mathematical Economics in Honor of Oskar Morgenstern, M. Shubik, ed., Princeton University Press, Princeton, N.J., 1967, pp. 39-52.

12. C.F. Doran, "The OPEC strategy space: Issues of conflict and cohesion", Unpublished Manuscript, Rice University, August, 1975.

13. Federal Energy Administration, Project Independence Report, Washington, D.C., U.S. Govt. Printing Office Stock No. 4118-00029, November, 1974.

14. D. Fisher, D. Gately and J.F. Kyle, "The prospects for OPEC: A critical survey of models of the world oil market", Discussion Paper Series No. 75-38, Center for Applied Economics, New York University, N.Y., July, 1975.

15. D.B. Gillies, "Solutions to general non-zero-sum games, "Contributions to the Theory of Games, Vol. IV, A.W. Tucker and R.D. Luce, eds., Annals of Math. Studies No. 40, Princeton University Press, Princeton, N.J., 1959, pp. 47-85.

16. J.H. Grotte, "Computation of and observations on the nucleolus, the normalized nucleolus, and the central games", M.S. Thesis, Center for Applied Mathematics, Cornell University, 1970.

17. J.H. Hayes, "Petroleum - A prediction made in 1950", Rand Paper No. P-5135, The Rand Corpn., Santa Monica, Calif. December, 1973.

18. W.W. Hogan, "Energy policy models for Project Independence", Comp. and Operations Research, 2, 1975, pp. 251-271.

19. A.L. Horelick, "The Soviet Union, the middle east and the evolving world energy situation", Rand Paper P-5109, The Rand Corporation, Santa Monica, Calif., October, 1973.

20. W.F. Lucas, "On mathematics in energy research", in Energy: Mathematics and Models, ed. by F.S. Roberts, SIAM Publications, Philadelphia, 1976, pp. 253-263.

21. H. Mendershausen, R.D. Nehring, "Protecting the U.S. petroleum market against future denials of imports, "Rand Report No. R-1603-ARPA, The Rand Corpn., Santa Monica, Calif., October, 1974.

22. L. Moore, "A game theoretic approach to the OPEC cartel", Unpublished Manuscript, OASIA, Treasury Department, Washington, D.C., August, 1975.

23. J. von Neumann and O. Morgenstern, Theory of Games and Economic Behaviour, Princeton University Press, Princeton, N.J., 1st edition 1944, 2nd edition, 1947, 3rd edition 1953.

24. B. Peleg, "Bargaining sets of cooperative games without side payments", Israel J. Math., 1, 1963, pp. 197-200.

25. B. Peleg, "Existence theorem for the bargaining set", Essays in Mathematical Economics in Honor of Oskar Morgenstern, M. Shubik, ed., Princeton University Press, Princeton, N.J., 1967, pp. 53-56.

26. H. Scarf, "The core of an n-person game", Econometrica, 35, 1967, pp. 60-69.

27. D. Schmeidler, "The nucleolus of a characteristic function game", SIAM J. Appl. Math., 17, 1969, pp. 1163-1170.

28. L.S. Shapley, "A value for n-person games", Contribution to the Theory of Games, Vol. II, H.W. Kuhn and A.W. Tucker, eds., Annals of Math. Studies No. 28, Princeton University Press, Princeton, N.J., 1953, pp. 307-317.

29. P.P. Shenoy, "A 2-person non-zero-sum game model of the world oil market", Technical Report No. 352, School of Operations Research and Industrial Engineering, Cornell University, Ithaca, N.Y., August 1977.

30. B.A. Smith, T.F. Sparrow, "Complementarity and substitutability in energy systems models", Unpublished Manuscript, Dept. of Economics, University of Houston, Houston, Texas, August, 1975.

31. R.M. Thrall, C. Doran, G. Owen, F.T. Wall and R.D. Young, "Roles for game theory in energy planning and management", in Strategic Studies for Energy Planning and Management, A Final Report on N.S.F. Grant NSF SIA75-04329, Rice University, Houston, Texas, June, 1976.

32. F.T. Wall, "Maximization of total profits attending exploitation of finite resources", in Strategic Studies for Energy Planning and Management, A Final Report on N.S.F. Grant NSF SIA75-04329, Rice University, Houston, Texas, June, 1976.

33. A.W. Wortham, "Talking points for workshop on game-theoretic analysis in energy policy problems, Unpublished Manuscript, Rice University, August, 1975.

34. R.D. Young, "A note on intertemporal game models of energy, exchange and economic growth", in Strategic Studies for Energy Planning and Management, A Final Report on N.S.F. Grant NSF SIA75-04329, Rice University, Houston, Texas, June, 1976.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER

Technical Report No. 357

2. ~~GOVT ACCESSION NO.~~

(14) TR-357

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)

(6) A 3-PERSON COOPERATIVE GAME IN CHARACTERISTIC FUNCTION FORM FORMULATION OF THE WORLD OIL MARKET.

5. TYPE OF REPORT & PERIOD COVERED

(9) Technical Report.

6. PERFORMING ORG. REPORT NUMBER

Technical Report No. 357

7. AUTHOR(s)

(10) Prakash P. Shenoy

8. CONTRACT OR GRANT NUMBER(s)

(15) N00014-75-C-0678, NSF-MPS-75-06028

9. PERFORMING ORGANIZATION NAME AND ADDRESS

School of Operations Research and Industrial Engineering, Cornell University, Upson Hall, Ithaca, New York 14853

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS

Operations Research Program
Office of Naval Research
Arlington, VA 22217

12. REPORT DATE

(11) August 1977

13. NUMBER OF PAGES

26 (12) 31p.

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report)

UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

409 869

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Energy
World Oil Market
Characteristic function game
Games with side payments
Games without side payments
Core
Shapley value
Bargaining set
Nucleolus

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The world oil market is modelled as a 3-person cooperative game in characteristic function form with and without side payments. The three players are the oil importing countries, the group in OPEC led by Iran and the group in OPEC led by Saudi Arabia. Various solutions of the side payment game and the non side payment game are studied.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

bpg